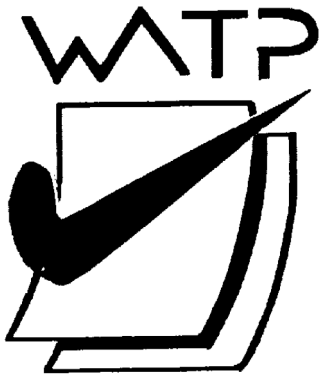


Semester Two Examination 2016

Question/Answer Booklet

MATHEMATICS METHODS UNITS 3 & 4

Section One: Calculator-free

Student Name: _____

Teacher's Name: _____

Time allowed for this section

Reading time before commencing work: five minutes
Working time for paper: fifty minutes

Material required/recommended for this section

To be provided by the supervisor
This Question/Answer booklet
Formula Sheet

To be provided by the candidate
Standard items: pens(blue/black preferred), pencils(including coloured), sharpener, correction tape/fluid, erasers, ruler, highlighters

Special Items: nil

Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised notes or other items of a non-personal nature in the examination room. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Structure of this paper

	Number of questions available	Number of questions to be attempted	Working time (minutes)	Marks available	Percentage of exam
Section One Calculator—free	**8**	**8**	**50**	**50**	**35**
Section Two Calculator—assumed	13	13	100	100	65
					100

Instructions to candidates

1. The rules for the conduct of Western Australian external examinations are detailed in the *Year 12 Information Handbook 2016.* Sitting this examination implies that you agree to abide by these rules.

2. Answer the questions according to the following instructions.

   Section One: Write answers in this Question/Answer Booklet. Answer **all** questions.

   Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat an answer to any question, ensure that you cancel the answer you do not wish to have marked.

   It is recommended that you **do not use pencil**, except in diagrams.

3. You must be careful to confine your responses to the specific questions asked and to follow any instructions that are specific to a particular question.

4. Spare pages are included at the end of this booklet. They can be used for planning your responses and/or as additional space if required to continue an answer.
   - Planning: If you use the spare pages for planning, indicate this clearly at the top of the page.
   - Continuing an answer: If you need to use the space to continue an answer, indicate in the original answer space where the answer is continued, i.e. give the page number. Fill in the number of the question that you are continuing to answer at the top of the page.

5. The Formula Sheet is **not** handed in with your Question/Answer Booklet.

See next page

Section One: Calculator–free 50 marks

This section has **eight (8)** questions. Attempt **all** questions. Write your answers in the spaces provided.

Spare pages are included at the end of this booklet. They can be used for planning your responses and/or as additional space if required to continue an answer.

- Planning: If you use the spare pages for planning, indicate this clearly at the top of the page.
- Continuing an answer: If you need to use the space to continue an answer, indicate in the original answer space where the answer is continued, i.e. give the page number. Fill in the number of the question(s) that you are continuing to answer at the top of the page.

Working time: 50 minutes

Question 1 (6 marks)

Determine each of the following, leaving your answers with positive indices. Do not simplify.

(a) $\frac{d}{dx}\left(\ln(2x+1) - \frac{2}{x^2}\right)$ (2 marks)

(b) $\frac{dx}{dt}$ where $x = \frac{e^{2t} - e^{t}}{2}$ (2 marks)

(c) $f'(y)$ where $f(y) = \sin 3y + 2\cos(1-2y)$ (2 marks)

See next page

Question 2 (7 marks)

Solve each of the following equations, leaving your answers in exact form.

(a) $3^{x+2} = 6$ (2 marks)

(b) $\ln x = \ln x^2 + 2$ (3 marks)

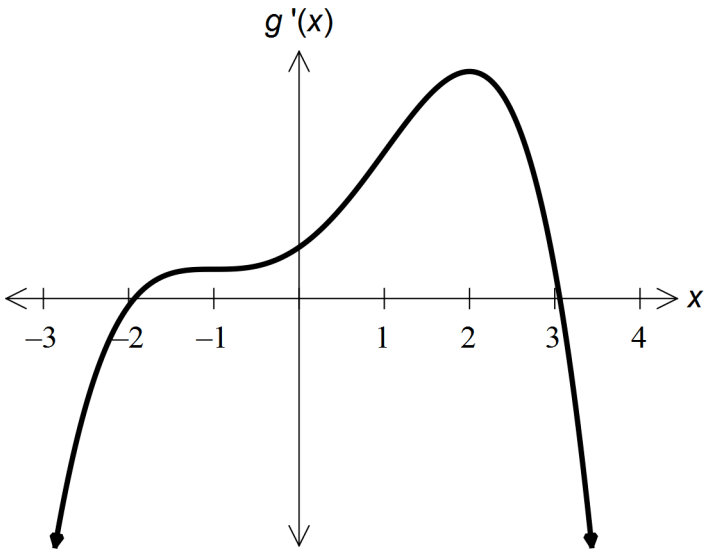
(c) $\frac{d}{dx}\left(\frac{e^{\sqrt{x}} + e^x}{2}\right) = e^x$ (2 marks)

See next page

Question 3 (7 marks)

(a) Determine the minimum value of the function $f(x) = x^2 \ln x$.
Use the second derivative test to prove that your value is a minimum. (4 marks)

(b) The graph of $y = g'(x)$ is sketched below.
On the same axes, sketch $y = g''(x)$. (3 marks)



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Question 4 (7 marks)

Determine the following, giving your answers in exact form.

(a) $\int \left(\frac{2}{x} + \sin\left(\frac{x}{2} + 3 \right) \right) dx$ (2 marks)

(b) $\int_{-1}^{0} (x^2 - e^{x+1})\, dx$ (3 marks)

(c) $\frac{d}{dx}\left(\int_{2x}^{e} \tan x \ dx \right)$ (2 marks)

See next page

Question 5 (7 marks)

The function $f(x)$ is defined as:

$$f(x) = \begin{cases} \frac{x}{6} & x = 0,1 \\ \frac{1}{x} & x = 2,3 \\ 0 & \text{otherwise} \end{cases}$$

(a) Explain why x is a discrete random variable. (1 mark)

(b) Show clearly that $f(x)$ defines a probability function. (2 marks)

(c) Determine:

(i) $P(x = 2 \cup x = 1)$ (1 mark)

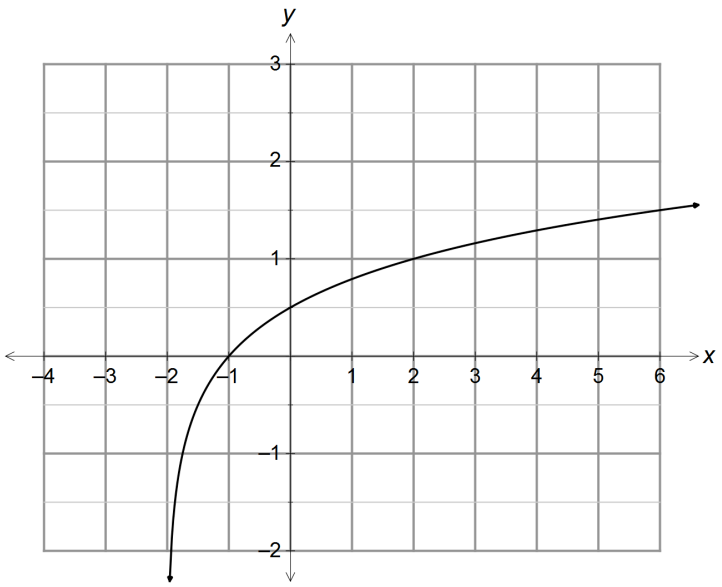
(ii) $P(x < 2)$ (1 mark)

(iii) $P(x \geq 2 \mid x < 3)$ (2 marks)

See next page

Question 6 (6 marks)

The graph $y = \log_b (x + c)$ is sketched below.



(a) State the values of b and c. (2 marks)

(b) Sketch the following graphs on the same axes.

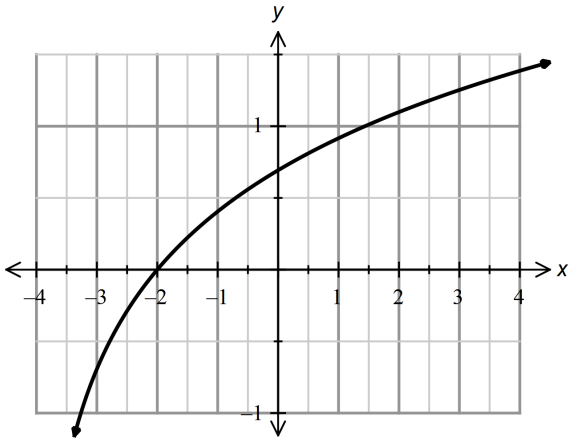
(i) $f(x) = \log_b (x - c)$. (2 marks)

(ii) $g(x) = \log_b (c - x)$. (2 marks)

See next page

Question 7 (5 marks)

The graph of $y = \ln\left(\frac{x}{2} + 2\right)$ is shown below.



(a) Estimate, to the nearest unit^2, the area bounded by the curve, the x–axis and the line $x = 2$. Show your reasoning clearly. (2 marks)

Hence, or otherwise:

(b) estimate the value of:

(i) $\int_{-2}^{2} \ln\left(\frac{x}{2} + 2\right) dx$ (1 mark)

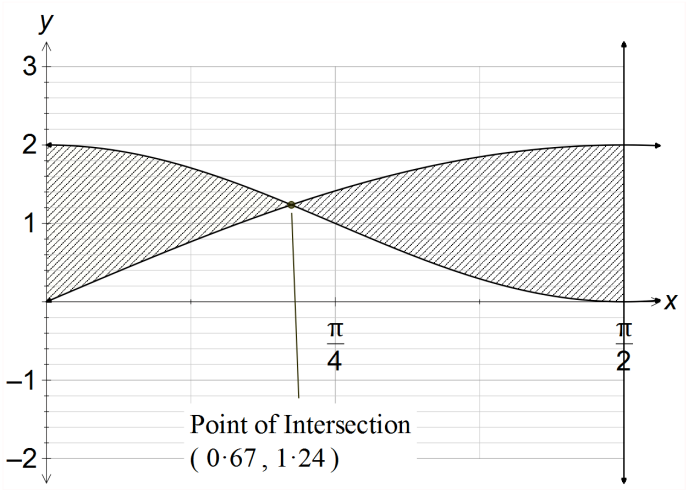
(ii) $\int_{-2}^{2} \ln\left(\frac{x}{2} + 2\right)^{-1} dx$ (1 mark)

(iii) $\int_{-5}^{-1} \ln\left(\frac{x}{2} + 5\right)^{-1} dx$ (1 mark)

See next page

Question 8 (5 marks)

The shaded region below is the area between the curves y = 2sin x and $y = 1 + \cos 2x$, between $x = 0$ and $x = \frac{\pi}{2}$.



State the integrals required to find the following areas. Do not evaluate them.

(a) The area bounded by the curve y = 2sin x , the line $x = \frac{\pi}{2}$ and the x–axis. (2 marks)

(b) The area bounded by curves $y = 2\sin x$ and $y = 1 + \cos 2x$ between $x = 0$ and the line $x = \frac{\pi}{2}$. (3 marks)

End of Section One

Additional working space

Question number(s): ……